Chapter five

Equations with One Variable

We have known in the previous class what equation is and learnt its usage. We have learnt the solution of simple equations with one variable and acquired knowledge thoroughly about the solution of simple equations by forming equations from real life problems. In this chapter, linear and quadratic equations and Identities have been discussed and their usages have been shown to solve the real life problems.

At the end of the chapter, the students will be able to –

- Explain the conception of variable
- Explain the difference between equation and identity
- Solve the linear equations
- Solve by forming linear equations based on real life problems
- Solve the quadratic equations and find the solution sets
- Form the quadratic equations based on real life problems and solve.

5·1 Variables

We know, $x+3=5$ is an equation. To solve it, we find the value of the unknown quantity x. Here the unknown quantity x is a variable. Again, to solve the equation $x+a=5$, we find the value of x, not the value of a. Here, x is assumed as variable and a as constant. In this case, we shall get the values of x in terms of a. But if we determine the value of a, we shall write $a=5-x$; that is, the value of a will be obtained in terms of x. Here a is considered a variable and x a constant. But if no direction is given, conventionally x is considered a variable. Generally, the small letters x, y, z, the ending part of English alphabet are taken as variables and a, b, c, the starting part of the alphabet are taken as constants.

The equation, which contains only one variable, is called a linear equation with one variable. Such as, in the equation $x+3=5$, there is only one variable x. So this is the linear equation with one variable.

We know what the set is. If a set $S=\{x : x \in R, 1 \leq x \leq 10\}$, x may be any real number from 1 to 10. Here, x is a variable. So, we can say that when a letter symbol means the element of a set, it is called variable.

Degree of an equation : The highest degree of a variable in any equation is called the degree of the equation. Degree of each of the equations $x+1=5,\ 2x-1=x+5,\ y+7=2y-3$ is 1 ; these are linear equations with one variable.

Again, the degree of each of the equations $x^2+5x+6=0$, $y^2-y=12$, $4x^2-2x=3-6x$ is 2 ; these are quadratic equations with one variable. The equation $2x^3-x^2-4x+4=0$ is the equation of degree 3 with one variable.

5·2 Equation and Identity

Equation : There are two polynomials on two sides of the equal sign of an equation, or there may be zero on one side (mainly on right hand side). Degree of the variable of the polynomials on two sides may not be equal. Solving an equation, we get the number of values of the variable equal to the highest degree of that variable. This value or these values are called the roots of the equation. The equation will be satisfied by the root or roots. In the case of more than one root, these may be equal or unequal. Such as, roots of $x^2-5x+6=0$ are 2 an 3. Again, though the value of x in the equations $(x-3)^2=0$ is 3, the roots of the equation are 3, 3.

Identity` : There are two polynomials of same (equal) degree on two sides of equal sign. Identity will be satisfied by more values than the number of highest degree of the variable. There is no difference between the two sides of equal sign ; that is why, it is called identity. Such as, $(x+1)^2-(x-1)^2=4x$ is an identity ; it will be satisfied for all values of x. So this equation is an identity. Each algebraic formula is an identity. Such as, $(a+b)^2=a^2+2ab+b^2$, $(a-b)^2=a^2-2ab+b^2$, $a^2-b^2=(a+b)(a-b)$, $(a+b)^3=a^3+3a^2b+3ab^2+b^3$ etc. are identities.

All equations are not identities, In identity '≡' sign is used instead of equal (=) sign. But as all identities are equations, in the case of identity also, generally the equal sign is used. Distinctions between equation and identity are given below :

Equation	Identity
1. Two polynomials may exist on both sides of equal sign, or there may be zero on one side.	1. Two polynomials exist on two sides.
2. Degree of the polynomials on both sides may be unequal.	2. Degree of the polynomials on both sides is equal.
3. The equality is true for one or more values of the variable.	3. Generally, the equality is true for all values of the original set of the variable.
4. The number of values of the variable does not exceed the highest degree of the equation	4. Equality is true for infinite number of values of the variable.
5. All equations are not formulae.	5. All algebraic formulae are identities.

Activity :	1. What is the degree of and how many roots has each of the following equations ? (i) $3x+1=5$ (ii) $\frac{2y}{5}-\frac{y-1}{3}=\frac{3y}{2}$ 2. Write down three identities.

5·3 Solution of the equations of first degree

In case of solving equations, some rules are to be applied. If the rules are known, solution of equations becomes easier. The rules are as follows :

1. If the same number or quantity is added to both sides of an equation, two sides remain equal.
2. If the same number or quantity is subtracted from both sides of an equation, two sides remain equal.
3. If both sides of an equation are multiplied by the same number or quantity, the two sides remain equal.
4. If both sides of an equation are divided by same nonzero number or quantity, the two sides remain equal.

The rules stated above may be expressed in terms of algebraic expressions as follows:

If $x=a$ and $c\neq 0$, (i) $x+c=a+c$ (ii) $x-c=a-c$ (iii) $xc=ac$ (iv) $\frac{x}{c}=\frac{a}{c}$

Besides, if a, b and c are three quantities, if $a=b+c$, $a-b=c$ and if $a+c=b$, $a=b-c$.

This law is known as transposition law and different equations can be solved by applying this law.

If the terms of an equation are in fractional form and if the degree of the variables in each numerator is 1 and the denominator in each term is constant, such equations are linear equations.

Example 1. Solve : $\frac{5x}{7}-\frac{4}{5}=\frac{x}{5}-\frac{2}{7}$

Solution : $\frac{5x}{7}-\frac{4}{5}=\frac{x}{5}-\frac{2}{7}$ or, $\frac{5x}{7}-\frac{x}{5}=\frac{4}{5}-\frac{2}{7}$ [by transposition]

or, $\frac{25x-7x}{35}=\frac{28-10}{35}$ or, $\frac{18x}{35}=\frac{18}{35}$

or, $18x=18$

or, $x=1$

$\therefore$ Solution is $x=1$.

Now, we shall solve such equations which are in quadratic form. These equations are transformed into their equivalent equations by simplifications and lastly the equations is transformed into linear equation of the form $ax=b$. Again, even if there are variables in the denominator, they are also transformed into linear equation by simplification.

Example 2. Solve : $(y-1)(y+2)=(y+4)(y-2)$

Solution : $(y-1)(y+2)=(y+4)(y-2)$

or, $y^2-y+2y-2=y^2+4y-2y-8$

or, $y-2=2y-8$

or, $y-2y=-8+2$ [by transposition]

or, $-y=-6$

or, $y=6$

$\therefore$ Solution is $y=6$

Example 3. Solve and write the solution set : $\frac{6x+1}{15}-\frac{2x-4}{7x-1}=\frac{2x-1}{5}$

Solution : $\frac{6x+1}{15}-\frac{2x-4}{7x-1}=\frac{2x-1}{5}$

or, $\frac{6x+1}{15}-\frac{2x-1}{5}=\frac{2x-4}{7x-1}$ [by transposition]

or, $\frac{6x+1-6x+3}{15}=\frac{2x-4}{7x-1}$ or, $\frac{4}{15}=\frac{2x-4}{7x-1}$

or, $15(2x-4)=4(7x-1)$ [by cross-multiplication]

or, $30x-60=28x-4$

or, $30x-28x=60-4$ [by transposition]

or, $2x=56$, or $x=28$

$\therefore$ Solution is $x=28$

and solution set is $S=\{28\}$.

Example 4. Solve : $\frac{1}{x-3}+\frac{1}{x-4}=\frac{1}{x-2}+\frac{1}{x-5}$

Solution : $\frac{1`}{x-3}+\frac{1}{x-4}=\frac{1}{x-2}+\frac{1}{x-5}$

or, $\frac{x-4+x-3}{(x-3)(x-4)}=\frac{x-5+x-2}{(x-2)(x-5)}$ or, $\frac{2x-7}{x^2-7x+12}=\frac{2x-7}{x^2-7x+10}$

Values of the fractions of two sides are equal. Again, numerators of two sides are equal, but denominators are unequal. In this case, if only the value of the numerators is zero, two sides will be equal.

$\therefore$ $2x-7=0$ or, $2x=7$

or, $x=\frac{7}{2}$

$\therefore$ Solution is $x=\frac{7}{2}$

Example 5. Find the solution set : $\sqrt{2x-3}+5=2$

Solution : $\sqrt{2x-3}+5=2$

or, $\sqrt{2x-3}=2-5$ [by transposition]

or, $\left(\sqrt{2x-3}\right)^2=(-3)^2$ [squaring both sides]

or, $2x-3=9$

or, $2x=12$

or, $x=6$

Since there is the sign of square root, verification of the correctness is necessary.

Putting $x=6$ in the given equation, we get,

$\sqrt{2\times 6-3}+5=2$ or $\sqrt{9}+5=2$

or, $3+5=2$

or, $8=2$, which is impossible.

$\therefore$ The equation has no solution.

$\therefore$ Solution set is :S = ϕ

Alternative method :

$\sqrt{2x-3}+5=2$

or, $\sqrt{2x-3}=2-5$

or, $\sqrt{2x-3}=-3$

Square root of any real quantity cannot be negative.

$\therefore$ The equation has no solution.

$\therefore$ Solution set is :S = ϕ

Activities : 1. If $\left(\sqrt{5}+1\right)x+4=4\sqrt{5}$, show that, $x=6-2\sqrt{5}$

2. Solve and write the solution set : $\sqrt{4x-3}+5=2$

5·4 Usage of linear equations

In real life we have to solve different types of problems. In most cases of solving these problems mathematical knowledge, skill and logic are necessary. In real cases, in the application of mathematical knowledge and skill, as on one side the problems are solved smoothly, on the other side in daily life, solutions of the problems are obtained by mathematics. As a result, the students are interested in mathematics. Here different types of problems based on real life will be expressed by equations and they will be solved.

For determining the unknown quantity in solving the problems based on real life, variable is assumed instead of the unknown quantity and then equation is formed by the given conditions. Then by solving the equation, value of the variable, that is the unknown quantity is found.

Example 6. The digit of the units place of a number consisting of two digits is 2 more than the digit of its tens place. If the places of the digits are interchanged, the number thus formed will be less by 6 than twice the given number. Find the number.

Solution : Let the digit of tens place be x. Then the digit of units place will be $x+2$.

$\therefore$ the number is $10x+(x+2)$ or, $11x+2$.

Now, if the places of the digits are interchanged, the changed number will be $10(x+2)+x$ or $11x+20$

By the question, $11x+20=2(11x+2)-6$
or, $11x+20=22x+4-6$
or, $22x-11x=20+6-4$ [by transposition]
or, $11x=22$
or, $x=2$
$\therefore$ The number is $11x+2=11\times2+2=24$
$\therefore$ given number is 24.

Example 7. In a class if 4 students are seated in each bench, 3 benches remain vacant. But if 3 students are seated on each bench, 6 students are to remain standing. What is the number of students in that class ?

Solution : Let the number of students in the class be x.
Since, if 4 students are seated in a bench, 3 benches remain vacant, the number of benches of that class $=\frac{x}{4}+3$
Again, since, if 3 students are seated in each bench, 6 students are to remain standing, the number of benches of that class $=\frac{x-6}{3}$
Since the number of benches is fixed,
$\frac{x}{4}+3=\frac{x-6}{3}$ or, $\frac{x+12}{4}=\frac{x-6}{3}$
or, $4x-24=3x+36$, or, $4x-3x=36+24$
or, $x=60$
$\therefore$ number of students of the class is 60.

Example 8. Mr. Kabir, from his Tk. 56000, invested some money at the rate of profit 12% per annum and the rest of the money at the rate of profit 10% per annum. After one year he got the total profit of Tk. 6400. How much money did he invest at the rate of profit 12% ?

Solution : Let Mr. Kabir invest Tk. x at the rate of profit 12%.
$\therefore$ he invested Tk. $(56000-x)$ at the rate of profit 10%.
Now, profit of Tk. x in 1 year is Tk. $x\times\frac{12}{100}\times1$, or, Tk. $\frac{12x}{100}$
Again, profit of Tk. $(56000-x)$ in 1 year is Tk. $(56000-x)\times\frac{10}{100}$,
or, Tk. $\frac{10(56000-x)}{100}$
By the question, $\frac{12x}{100}+\frac{10(56000-x)}{100}=6400$
or, $12x+560000-10x=640000$
or, $2x=640000-560000$

or, $2x = 80000$
or, $x = 40000$
$\therefore$ Mr. Kabir invested Tk. 40000 at the rate of profit 12%.

Activity : Solve by forming equations :

1. What is the number, if any same number is added to both numerator and denominator of the fraction $\frac{3}{5}$, the fraction will be $\frac{4}{5}$?
2. If the difference of the squares of two consecutive natural numbers is 151, find the two numbers.
3. If 120 coins of Tk. 1 and Tk. 2 together are Tk. 180, what is the number of coins of each kind ?

Exercise 5·1

Solve (1-10) :

1. $3(5x-3) = 2(x+2)$
2. $\frac{ay}{b} - \frac{by}{a} = a^2 - b^2$
3. $(z+1)(z-2) = (z-4)(z+2)$
4. $\frac{7x}{3} + \frac{3}{5} = \frac{2x}{5} - \frac{4}{3}$
5. $\frac{4}{2x+1} + \frac{9}{3x+2} = \frac{25}{5x+4}$
6. $\frac{1}{x+1} + \frac{1}{x+4} = \frac{1}{x+2} + \frac{1}{x+3}$
7. $\frac{a}{x-a} + \frac{b}{x-b} = \frac{a+b}{x-a-b}$
8. $\frac{x-a}{b} + \frac{x-b}{a} + \frac{x-3a-3b}{a+b} = 0$
9. $\frac{x-a}{a^2-b^2} = \frac{x-b}{b^2-a^2}$
10. $(3+\sqrt{3})z + 2 = 5 + 3\sqrt{3}$.

Find the solution set (11 - 19) :

11. $2x(x+3) = 2x^2 + 12$
12. $2x + \sqrt{2} = 3x - 4 - 3\sqrt{2}$
13. $\frac{x+a}{x-b} = \frac{x+a}{x+c}$
14. $\frac{z-2}{z-1} = 2 - \frac{1}{z-1}$
15. $\frac{1}{x} + \frac{1}{x+1} = \frac{2}{x-1}$
16. $\frac{m}{m-x} + \frac{n}{n-x} = \frac{m+n}{m+n-x}$
17. $\frac{1}{x+2} + \frac{1}{x+5} = \frac{1}{x+4} + \frac{1}{x+3}$
18. $\frac{2t-6}{9} + \frac{15-2t}{12-5t} = \frac{4t-15}{18}$
19. $\frac{x+2b^2+c^2}{a+b} + \frac{x+2c^2+a^2}{b+c} + \frac{x+2a^2+b^2}{c+a} = 0$

Solve by forming equations (20 - 27) :

20. A number is $\frac{2}{5}$ times of another number. If the sum of the numbers is 98, find the two numbers.
21. Difference of num. and denom. of a proper fraction is 1. If 2 is subtracted from numerator and 2 is added to denominator of the fraction, it will be equal to $\frac{1}{6}$. Find the fraction.
22. Sum of the digits of a number consisting of two digits is 9. If the number obtained by interchanging the places of the digits is less by 45 than the given number, what is the number ?
23. The digit of the units place of a number consisting of two digits is twice the digit of the tens place. Show that, the number is seven times the sum of the digits.
24. A petty merchant by investing Tk. 5600 got the profit 5% on some of the money and profit of 4% on the rest of the money. On how much money did he get the profit of 5% ?
25. Number of passengers in a launch is 47 ; the fare per head for the cabin is twice that for the deck. The fare per head for the deck is Tk. 30. If the total fare collected is Tk. 1680, what is the number of passengers in the cabin ?
26. 120 coins of twenty five paisa and fifty paisa together is Tk. 35. What is the number of coins of each kind ?
27. A car passed over some distance at the speed of 60 km per hour and passed over the rest of the distance at the speed of 40 km per hour. The car passed over the total distance of 240 km in 5 hours. How far did the car pass over at the speed of 60 km per hour ?

5·5 Quadratic equation with one variable

Equations of the form $ax^2 + bx + c = 0$ [where a, b, c are constants and $a \neq 0$] is called the quadratic equation with one variable. Left hand side of a quadratic equation is a polynomial of second degree, right hand side is generally taken to be zero.

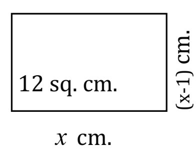
Length and breadth of a rectangular region of area 12 square cm. are respectively x cm. and $(x-1)$ cm.

$\therefore$ area of the rectangular region is = $x(x-1)$ square cm.

By the question, $x(x-1)=12$, or $x^2 - x - 12 = 0$.

x is the variable in the equation and highest power of x is 2.

Such equation is a quadratic equation. The equation, which has the highest degree 2 of the variable, is called the quadratic equation.



In class VIII, we have factorized the quadratic expressions with one variable of the forms $x^2 + px + q$ and $ax^2 + bx + c$. Here, we shall solve the equations of the forms $x^2 + px + q = 0$ and $ax^2 + bx + c = 0$ by factorizing the left hand side and by finding the value of the variable.

An important law of real numbers is applied to the method of factorization. The law is as follows :

If the product of two quantities is equal to zero, either ony one of the quantities or both quantities will be zero. That is, if the product of two quantities a and b i.e., $ab = 0$, $a = 0$ or, $b = 0$, or, both $a = 0$ and $b = 0$.

Example 9. Solve : $(x+2)(x-3) = 0$

Solution : $(x+2)(x-3) = 0$

$\therefore$ $x+2=0$, or, $x-3=0$

If $x+2=0$, $x=-2$

Again, if $x-3=0$, $x=3$

$\therefore$ solution is $x=-2$ or, 3.

Example 10. Find the solution set : $y^2 = \sqrt{3}y$

Solution : $y^2 = \sqrt{3}y$

or, $y^2 - \sqrt{3}y = 0$ [By transposition, right hand side has been done zero]

or, $y(y-\sqrt{3}) = 0$

$\therefore$ $y = 0$, or $y - \sqrt{3} = 0$

If $y-\sqrt{3} = 0$, $y = \sqrt{3}$

$\therefore$ Solution set is $\{0, \sqrt{3}\}$.

Example 11. Solve and write the solution set : $x - 4 = \dfrac{x-4}{x}, x \neq 0$.

Solution : $x - 4 = \dfrac{x-4}{x}$

or, $x(x-4) = x-4$ [by cross-multiplication]

or, $x(x-4)-(x-4) = 0$ [by transposition]

or, $(x-4)(x-1) = 0$

$\therefore$ $x-4=0$, or, $x-1=0$

If $x-4=0$, $x=4$

Again, if $x-1=0$, $x=1$

$\therefore$ Solution is : $x=1$ or, 4

and the solution set is $\{1, 4\}$.

Example 12. Solve : $\left(\dfrac{x+a}{x-a}\right)^2 - 5\left(\dfrac{x+a}{x-a}\right) + 6 = 0$

Solution : $\left(\dfrac{x+a}{x-a}\right)^2 - 5\left(\dfrac{x+a}{x-a}\right) + 6 = 0$..........(1)

Let, $\dfrac{x+a}{x-a} = y$

Then from (1), we get, $y^2 - 5y + 6 = 0$

or, $y^2 - 2y - 3y + 6 = 0$

or, $y(y-2)-3(y-2)=0$

or, $(y-2)(y-3)=0$

$\therefore$ $y-2=0$, or, $y-3=0$

If $y-2=0$, $y=2$

If $y-3=0$, $y=3$

Now, when $y=2$,

$\frac{x+a}{x-a}=\frac{2}{1}$ [putting the value of y]

or, $\frac{x+a+x-a}{x+a-x+a}=\frac{2+1}{2-1}$ [by componendo and dividendo]

or, $\frac{2x}{2a}=\frac{3}{1}$

or, $x=3a$

Again, when $y=3$, $\frac{x+a}{x-a}=\frac{3}{1}$

or, $\frac{x+a+x-a}{x+a-x+a}=\frac{3+1}{3-1}$

or, $\frac{2x}{2a}=\frac{4}{2}$

or, $\frac{x}{a}=\frac{2}{1}$

or, $x=2a$

$\therefore$ Solution is : $x=3a$, or, $2a$

Activity :

1. Comparing the equation $x^2-1=0$ with the equation $ax^2+bx+c=0$, write down the values of a,b,c.
2. What is the degree of the equation $(x-1)^2=0$? How many roots has the equation and what are they?

5·6 Usage of quadratic equations

Many problems of our daily life can be solved easily by forming linear and quadratic equations. Here, the formation of quadratic equations from the given conditions based on real life problems and techniques for solving them are discussed.

Example 13. Denominator of a proper fraction is 4 more than the numerator. If the fraction is squared, its denominator will be 40 more than the numerator. Find the fraction.

Solution : Let the fraction be $\frac{x}{x+4}$.

Square of the fraction $= \left(\dfrac{x}{x+4}\right)^2 = \dfrac{x^2}{(x+4)^2} = \dfrac{x^2}{x^2+8x+16}$

Here, numerator $= x^2$ and denominator $= x^2+8x+16$.

By the question, $x^2+8x+16 = x^2+40$

or, $8x+16 = 40$

or, $8x = 40-16$

or, $8x = 24$

or, $x = 3$

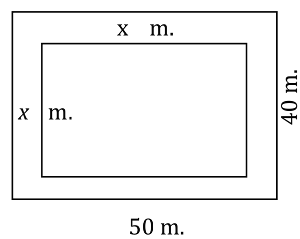
$\therefore\ x+4 = 3+4 = 7$

$\therefore\ \dfrac{x}{x+4} = \dfrac{3}{3+4} = \dfrac{3}{7}$

$\therefore$ the fraction is $\dfrac{3}{7}$

Example 14. A rectangular garden with length 50 metre and breadth 40 metre has a path of equal width all around the inside of the garden. If the area of the garden except the path is 1200 square metre, how much is the path wide in metre ?

Solution : Let the path be x metre wide.



Without the path,

Length of the garden $= (50-2x)$ metre and

its breadth $= (40-2x)$ metre

$\therefore$ Without the path, area of the garden $= (50-2x) \times (40-2x)$ square metre.

By the question, $(50-2x)(40-2x) = 1200$

or, $2000 - 80x - 100x + 4x^2 = 1200$

or, $4x^2 - 180x + 800 = 0$

or, $x^2 - 45x + 200 = 0$ [dividing by 4]

or, $x^2 - 5x - 40x + 200 = 0$

or, $x(x-5) - 40(x-5) = 0$

or, $(x-5)(x-40) = 0$

$\therefore\ x-5 = 0$, or $x-40 = 0$

If $x-5 = 0$, $x = 5$

If $x-40 = 0$, $x = 40$

But the breadth of the path will be less than 40 metre from the breadth of the garden.

$\therefore\ x \neq 40$; $\therefore\ x = 5$

$\therefore$ the path is 5 metres wide.

Example 15. Shahik bought some pens at Tk. 240. If he would get one more pen in that money, average cost of each pen would be less by Tk. 1. How many pens did he buy ?

Solution : Let, Shahik bought x pens in total by Tk. 240. Then each pen costs Tk. $\frac{240}{x}$. If he would get one more pen, that is, he would get $(x+1)$ pens, the cost of each pen would be Tk. $\frac{240}{x+1}$.

By the question, , $\frac{240}{x+1}=\frac{240}{x}-1$, or $\frac{240}{x+1}=\frac{240-x}{x}$

or, $240x=(x+1)(240-x)$ [by cross-multiplication]

or, $240x=240x+240-x^2-x$

or, $x^2+x-240=0$ [by transposition]

or, $x^2+16x-15x-240=0$

or, $x(x+16)-15(x+16)=0$

or, $(x+16)(x-15)=0$

$\therefore x+16=0$, or, $x-15=0$

If $x+16=0$, $x=-16$

Again, if $x-15=0$, $x=15$

But the number of pen x, cannot be negative.

$\therefore\ x\neq-16$; $\therefore\ x=15$

$\therefore$ Shahik bought 15 pens.

Activity : Solve by forming equations :

1. If a natural number is added to its square, the sum will be equal to nine times of exactly its next natural number. What is the number ?
2. Length of a perpendicular drawn from the centre of a circle of radius 10 cm. to a chord is less by 2 cm. than the semi-chord. Find the length of the chord by drawing a probable picture.

Example 16. In an examination of class IX of a school, total marks of x students obtained in mathematics is 1100. If at the same examination, marks of a new student in mathematics is 34 and it is added to the former total marks, the average of the marks become less by 1.

a. Write down the average of the obtained marks of all students including the new student and separately x students in terms of x.

b. By forming equation from the given conditions, show that, $x^2+35x-1100=0$

c. By finding the value of x, find the average of the marks in the two cases.

Solution : a. Average of the marks obtained by x students = $\frac{1100}{x}$

Average of the marks obtained by $(x+1)$ students including

the new student = $\dfrac{1950+34}{x+1}=\dfrac{1984}{x+1}$

b. By the question, $\dfrac{1950}{x}=\dfrac{1984}{x+1}+1$

or, $\dfrac{1950}{x}-\dfrac{1984}{x+1}=1$

or, $\dfrac{1950x+1950-1984x}{x(x+1)}=1$

or, $x^2+x=1950x-1984x+1950$ [by cross-multiplication]

or, $x^2+x=1950-34x$

$\therefore$ $x^2+35x-1950=0$ [showed]

c. $x^2+35x-1950=0$

or, $x^2+65x-30x-1950=0$

or, $x(x+65)-30(x+65)=0$

or, $(x+65)(x-30)=0$

$\therefore x+65=0$, or, $x-30=0$

If $x+65=0$, $x=-65$

Again, if $x-30=0$, $x=30$

Since the number of students, i.e., x cannot be negative, $x\neq-65$

$\therefore$ $x=30$.

$\therefore$ in the first case, average = $\dfrac{1950}{30}=65$

and in the second case, average = $\dfrac{1984}{31}=64$.

Exercise 5·2

1. Assuming x as the variable in the equation $a^2x+b=0$, which one of the following is the degree of the equation ?
   a. 3 b. 2 c. 1 d. 0
2. Which one of the following is an identity ?
   a. $(x+1)^2+(x-1)^2=4x$ b. $(x+1)^2+(x-1)^2=2(x^2+1)$
   c. $(a+b)^2-(a-b)^2=2ab$ d. $(a-b)^2=a^2+2ab+b^2$
3. How many roots are there in the equation $(x-4)^2=0$?
   a. 1 b. 2 c. 3 d. 4
4. Which one of the following are the two roots of the equation $x^2-x-12=0$?

5 What is the coefficient of x in the equation $3x^2 - x + 5 = 0$?

a. 3 b. 2 c. 1 d. -1

6 Observe the following equations :

i. $2x + 3 = 9$ *ii.* $\frac{x}{2} - 2 = -1$ *iii.* $2x + 1 = 5$

Which are of the above equations equivalent ?

a. *i* and *ii* b. *ii* and *iii* c. *i* and *iii* d. *i*, *ii* and *iii*

7 Which one of the following is the solution set of the equation $x^2 - (a+b)x + ab = 0$?

a. $\{a, b\}$ b. $\{a, -b\}$ c. $\{-a, b\}$ d. $\{-a, -b\}$

8 The digit of the tens place of a number consisting of two digits is twice the digit of the units place. In respect of the information, answer the following questions :

(1) If the digit of the units place is x, what is the number ?

a. $2x$ b. $3x$ c. $12x$ d. $21x$

(2) If the places of the digits are interchanged, what will be the number ?

a. $3x$ b. $4x$ c. $12x$ d. $21x$

(3) If $x = 2$, what will be the difference between the original number and the number by interchanging their places?

a. 18 b. 20 c. 34 d. 36

Solve (9–18) :

9 $(x+2)(x-\sqrt{3}) = 0$ 10. $(\sqrt{2}x + 3)(\sqrt{3}x - 2) = 0$ 11 $y(y-5) = 6$

12 $(y+5)(y-5) = 24$ 13. $2(z^2 - 9) + 9z = 0$ 14 $\frac{3}{2z+1} + \frac{4}{5z-1} = 2$

15 $\frac{4}{\sqrt{10x-4}} + \sqrt{10x-4} = 5$ 16 $\frac{x-2}{x+2} + \frac{6(x-2)}{x-6} = 1$ 17 $\frac{x}{a} + \frac{a}{x} = \frac{x}{b} + \frac{b}{x}$

18 $\frac{x-a}{x-b} + \frac{x-b}{x-a} = \frac{a}{b} + \frac{b}{a}$

Find the solution set (19–25):

19 $\frac{3}{x} + \frac{4}{x+1} = 2$ 20. $\frac{x+7}{x+1} + \frac{2x+6}{2x+1} = 5$ 21

$\frac{1}{x} + \frac{1}{a} + \frac{1}{b} = \frac{1}{x+a+b}$

22 $\frac{ax+b}{a+bx} = \frac{cx+d}{c+dx}$ 23. $x + \frac{1}{x} = 2$ 24 $2x^2 - 4ax = 0$

25. $\dfrac{(x+1)^3-(x-1)^3}{(x+1)^2-(x-1)^2}=2$

Solve by forming equations (26–31) :

26. Sum of the two digits of a number consisting of two digits is 15 and their product is 56, find the number.
27. Area of the floor of a rectangular room is 192 square metre. If the length of the floor is decreased by 4 metre and the breadth is increased by 4 metre, the area remains unchanged. Find the length and breadth of the floor.
28. Length of the hypotenuse of a right angled triangle is 15 cm. and the difference of the lengths of other two sides is 3 cm. Find the lengths of those two sides.
29. The base of a triangle is 6 cm. more than twice its height. If the area of the triangle is 810 square cm., what is its height ?
30. As many students are there in a class, each of them contributes equal to the number of class-mates of the class and thus total Tk. 420 was collected. What is the number of students in the class and how much did each student contribute ?
31. As many students are there in a class, each of them contributed 30 paisa more than the number of paisa equal to the number of students and thus total Tk. 70 was collected. What is the number of students in that class ?
32. Sum of the digits of a number consisting of two digits is 7. If the places of the digits are interchanged, the number so formed is 9 more than the given number.
    a. Write down the given number and the number obtained by interchanging their places in terms of variable x.
    b. Find the given number.
    c. If the digits of the original number indicate the length and breadth of a rectangular region in centimetre, find the length of its diagonal. Assuming the diagonal as the side of a square, find the length of the diagonal of the square.
33. The base and height of a right angle triangle are respectively $(x-1)$ cm. and x cm. and the length of the side of a square is equal to the height of the triangle. Again, the length of a rectangular region is $(x+3)$ cm. and its breadth is x cm.
    a. Show the information in only one picture.
    b. If the area of the triangular region is 10 square centimetre, what is its height?
    c. Find the successive ratio of the areas of the triangular, square and rectangular regions.